Ch. 4 Quadratic Equations
4.3 Solving Quad.Eq. by Completing the Square

Solving Quadratic Equations:

4.1 By graphing
4.2 By factoring
4.3 By completing the square
4.4 By using quadratic formula

EXAMPLE: Solve $x^2 - 4x - 5 = 0$

i) By factoring

$x^2-4x-5=0$ $\quad -5+1=-4$, $-5\times 1=-5$

$(x-5)(x+1)=0$

$x-5=0$ $\quad x+1=0$

$x_1=5$ $\quad x_2=-1$

ii) By completing the square

$0=x^2-4x-5$ $\quad b=-4$

$\left(\frac{b}{2}\right)^2=\left(\frac{-4}{2}\right)^2=(-2)^2=4$ $\quad +4$ $\quad -4$

$0=(x^2-4x+4)-4-5$

$0=(x-2)^2-9$

$+9 \quad +9$

$\pm\sqrt{9}=\sqrt{(x-2)^2}$

$\pm 3=x-2$

$+2 \quad +2$

$x_1=3+2=5$

$x_2=-3+2=-1$

* USE FACTORING WHENEVER POSSIBLE

Completing the Square (radical) method:

Best to use if the equation is already in **vertex form** or it **cannot be factored**:

1) Complete the square (if $b \neq 0$)
2) Isolate the squared term
3) Take square root of both sides
4) Make sure to keep both $\pm$ roots
5) *Discard extraneous root

↳ for WORD PROBLEMS where you'd have a restriction on a variable

1. $x^2 - 10 = 0$

$+10 \quad +10$

$\sqrt{x^2}=\pm\sqrt{10}$

$x=\pm\sqrt{10}$

$x_1=\sqrt{10}$

$x_2=-\sqrt{10}$

2. $(x-1)^2 - 49 = 0$

$+49 \quad +49$

$\sqrt{(x-1)^2}=\pm\sqrt{49}$

$x-1=\pm 7$

$+1 \quad +1$

$x_1=7+1=8$

$x_2=-7+1=-6$

3. $(x+2)^2 - 5 = 0$

$+5 \quad +5$

$\sqrt{(x+2)^2}=\pm\sqrt{5}$

$x+2=\pm\sqrt{5}$

$-2 \quad -2$

$x_1=\sqrt{5}-2$ OR $-2+\sqrt{5}$

$x_2=-\sqrt{5}-2$ OR $-2-\sqrt{5}$

4. $0 = x^2 + 4x - 10$

Complete the □

$0 = (x^2 + 4x + 4) - 4 - 10$

$0 = (x+2)^2 - 14$

$b = 4$ $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = (2)^2 = 4$ (+4, −4)

Solve for x

$0 = (x+2)^2 - 14$

$+14 \quad +14$

$\pm\sqrt{14} = \sqrt{(x+2)^2}$

$\pm\sqrt{14} = x + 2$

$-2 \quad -2$

$x_1 = \sqrt{14} - 2$

$x_2 = -\sqrt{14} - 2$

5. $5 = 2x^2 - 8x$

$5 = 2(x^2 - 4x)$ $b = -4$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$ (+4, −4)

$5 = 2([x^2 - 4x + 4] - 4)$

$5 = 2([x-2]^2 - 4)$

$5 = 2[x-2]^2 - 8$

$+8 \quad +8$

$\frac{13}{2} = \frac{2[x-2]^2}{2}$

$\pm\sqrt{\frac{13}{2}} = \sqrt{[x-2]^2}$

$\pm\sqrt{\frac{13}{2}} = x - 2$

$+2 \quad +2$

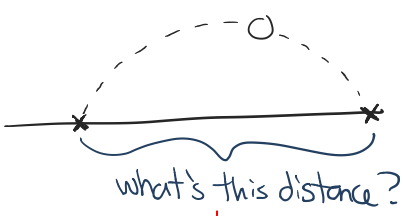
$x_1 = \sqrt{\frac{13}{2}} + 2$ $\qquad x_2 = -\sqrt{\frac{13}{2}} + 2$

Application Problem

How far does the soccer ball travel if the function that models its trajectory is as follows?

$h(x) = -0.016x^2 + 1.152x - 15.2$

h = height x = horizontal distance



$\frac{0}{-0.016} = \frac{-0.016x^2 + 1.152x - 15.2}{-0.016}$

$0 = x^2 - 72x + 950$ $b = -72$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-72}{2}\right)^2 = (-36)^2 = 1296$ (+1296, −1296)

$0 = (x^2 - 72x + 1296) - 1296 + 950$

$0 = (x-36)^2 - 346$

$+346 \quad +346$

Solve for x

$\pm\sqrt{346} = \sqrt{(x-36)^2}$

$\pm\sqrt{346} = x - 36$

$+36 \quad +36$

$x_1 = \sqrt{346} + 36 = 54.6$

$x_2 = -\sqrt{346} + 36 = 17.4$

$54.6 - 17.4 = 37.2$ units